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OPEN
10.1515/tvsb-2016-0013

Transactions of the VŠB – Technical University of Ostrava

Civil Engineering Series, Vol. 16, No. 2, 2016

paper #13

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SNAP-THROUGH OF THE VERY SHALLOW SHELL WITH INITIAL IMPERFECTION

Abstract

Elastic shallow shell of translation subjected to the external pressure is analysed in the paper numerically by FEM. Nonlinear equilibrium paths are calculated for the different boundary conditions. Special attention is paid to the influence of initial imperfection on the limit load level of fundamental load-displacement path of nonlinear analysis. ANSYS system was used for analysis, arc-length method was chosen for obtain fundamental load-displacement path of solution.

Keywords

Nonlinear stability, incremental solution, Newton-Raphson iteration, arc-length method.

1 INTRODUCTION

Shells of translation are structural elements very often encountered in the engineering practice. Their middle surface is generated by a vertical curve sliding along another vertical curve. The curves can be circles, ellipses, or parabolas. They occur as parts of aircraft and marine structures in mechanical engineering, create covers of large span structures in civil engineering.

These shells subjected to the external distributed load are liable to the buckling due to dominant compression membrane forces within the shell. It is the reason, why the stability problem has been analysed since the beginning of the twenty century. It was then when the first very slender structures of barrel shells appeared.

Solving stability of the thin shell, it is often insufficient to determine the elastic critical load from eigenvalue buckling analysis, i.e. the load, when perfect shell starts buckling. Nonlinear analysis is necessary, resulting in a full load-displacement response. Basis of this paper is to highlight the difference in the results of these two approaches. It is also necessary to include initial imperfections of real shell into the solution and determine limit load level more accurately. To confirm the high sensitivity of shallow shells to imperfections is also the aim of this paper. The geometrically nonlinear theory represents a basis for the reliable description of the postbuckling behaviour of the imperfect shell. Murray and Wilson [1] first presented idea of combining incremental (Euler) and iterative (Newton-Raphson) methods for solving nonlinear problems. Early works involving critical points and snap-through effect were written by Sharifi and Popov [2], and Sabir and Lock [3]. Using arc-length method to pass limit points on load-displacement paths introduced Riks in [4]. Getting through this problem using displacement control procedure presented Batoz and Dhatt [5]. Detection of critical points using arc-length method was introduced by Wriggers and Simo [6]. Works of Bathe [7] dominate in application of FEM to geometric nonlinear problems, Crisfield [8] incorporated problematic into pc codes.

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2 THEORY

Restricting to the isotropic elastic material and to the constant distribution of the residual stresses over the thickness, the total potential energy can be expressed as:

$$U = \int_A \frac{1}{2}(\boldsymbol{\varepsilon}_m - \boldsymbol{\varepsilon}_{0m})^T t\mathbf{D}(\boldsymbol{\varepsilon}_m - \boldsymbol{\varepsilon}_{0m})dA + \int_A \frac{1}{2}(\mathbf{k}-\mathbf{k}_0)^T \frac{t^3}{12}\mathbf{D}(\mathbf{k}-\mathbf{k}_0)dA - \int_A \mathbf{q}^T\mathbf{p}\,dA\,, \quad (1)$$

where $\boldsymbol{\varepsilon}_m, \boldsymbol{k}$ are strains and curvatures of the neutral surface, $\boldsymbol{\varepsilon}_{0m}, \boldsymbol{k}_0$ are initial strains and curvatures, $\boldsymbol{q}, \boldsymbol{p}$ are displacements of the point of the neutral surface, related load vector, $\boldsymbol{D}$ is the elasticity matrix.

The system of conditional equations [9] one can get from the condition of the minimum of the increment of the total potential energy $\delta\Delta U = 0$. This system can be written as:

$$\boldsymbol{K}_{inc}\,\Delta\boldsymbol{\alpha} + \boldsymbol{F}_{int} - \boldsymbol{F}_{ext} - \Delta\boldsymbol{F}_{ext} = \boldsymbol{0}\,, \quad (2)$$

where $\boldsymbol{K}_{inc}$ is the incremental stiffness matrix of shell, $\boldsymbol{F}_{int}$ are the internal forces of shell, $\boldsymbol{F}_{ext}$ is the external load of shell, $\Delta\boldsymbol{F}_{ext}$ is the increment of the external load of shell. Eq. (2) represents the base for the Newton-Raphson iteration and the incremental method as well.

In the case of the structure in equilibrium $\boldsymbol{F}_{int} - \boldsymbol{F}_{ext} = \boldsymbol{0}$, one can execute the incremental step $\boldsymbol{K}_{inc}\,\Delta\boldsymbol{\alpha} = \Delta\boldsymbol{F}_{ext} \Rightarrow \Delta\boldsymbol{\alpha} = \boldsymbol{K}_{inc}^{-1}\,\Delta\boldsymbol{F}_{ext}$ and $\boldsymbol{\alpha}^{i+1} = \boldsymbol{\alpha}^{i} + \Delta\boldsymbol{\alpha}$. The Newton-Raphson iteration can be arranged in the following way: supposing that $\boldsymbol{\alpha}^i$ does not represent exact solution, the residua are $\boldsymbol{F}_{\text{int}}^{i} - \boldsymbol{F}_{ext}^{i} = \boldsymbol{r}^i$. The corrected parameters are $\boldsymbol{\alpha}^{i+1} = \boldsymbol{\alpha}^{i} + \Delta\boldsymbol{\alpha}^{i}$, where $\Delta\boldsymbol{\alpha}^{i} = -\boldsymbol{K}_{inc}^{-1}\,\boldsymbol{r}^{i}$. The identity of the incremental stiffness matrix with the Jacobian of the system of the non-linear algebraic equation was used. Iteration process is finished using the suitable convergence norm.

3 FINITE ELEMENT ANALYSIS

Illustrative example of steel shallow shell loaded by the external pressure (Fig. 1) is presented. Results of eigenvalue buckling analysis are presented first. They offer an image about location of critical points of nonlinear solution, help with settings in the management of nonlinear calculation process. Results of fully nonlinear analysis follow (ideal shell and structure with initial imperfection).

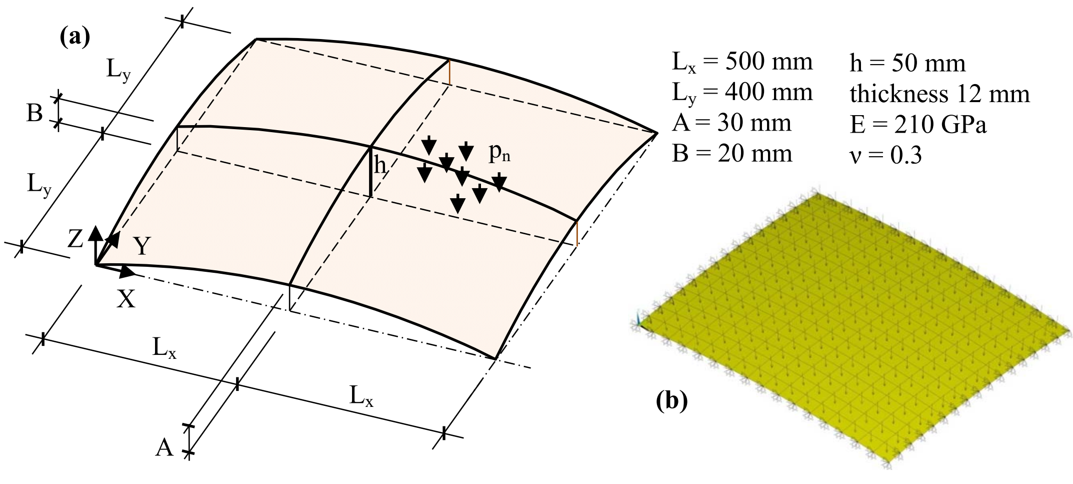

Fig. 1: Shallow shell of translation: parameters and numerical model

Presented results were obtained by division on 32x32 elements. Boundary conditions are first considered as simply supported along all edges (UX, UY and UZ applied on all lines), in the latter case shell is supported only at the corner (zero displacements are applied on corner nodes). Element type SHELL181 (4 nodes, 6 DOF at each node) was used [10]. The arc-length method was chosen for analysis, the reference arc-length radius is calculated from the load increment. Only fundamental path of nonlinear solution has been presented.

Consider firstly case 1 (shell simply supported along the edges). Results of eigenvalue buckling analysis are presented in Fig. 2. Elastic critical load i. e. the linearized stability problem of the eigenvalue and eigenvectors can be evaluated from

$$\left| \boldsymbol{K}_L - \lambda \boldsymbol{K}_G \right|_{\det} = 0\,, \tag{3}$$

where $\boldsymbol{K}_G$ is the geometric matrix (the matrix of increments of the bending stiffness due to action of the membrane forces), λ – the multiplier of the elastic critical load.

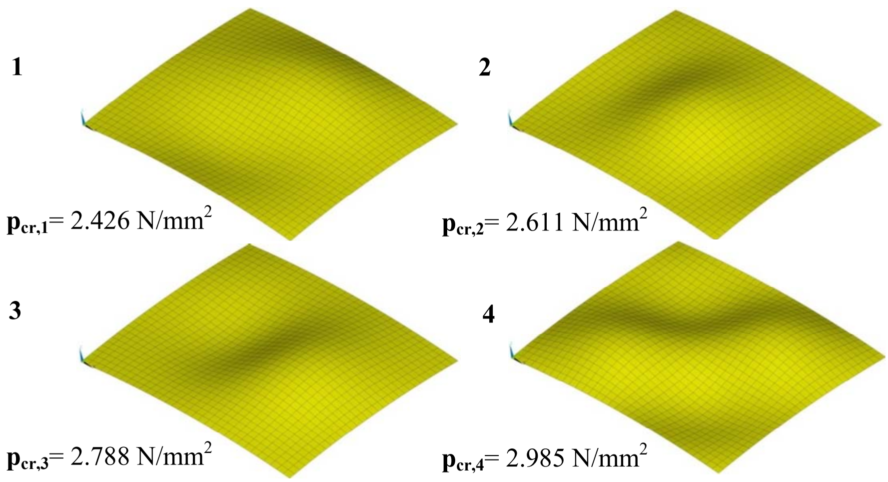

Fig. 2: Results of eigenvalue buckling analysis for case 1

The eigenvectors from the Eq. (3) represent the modes of buckling. First four critical load values and the modes of buckling are arranged in Fig. 2.

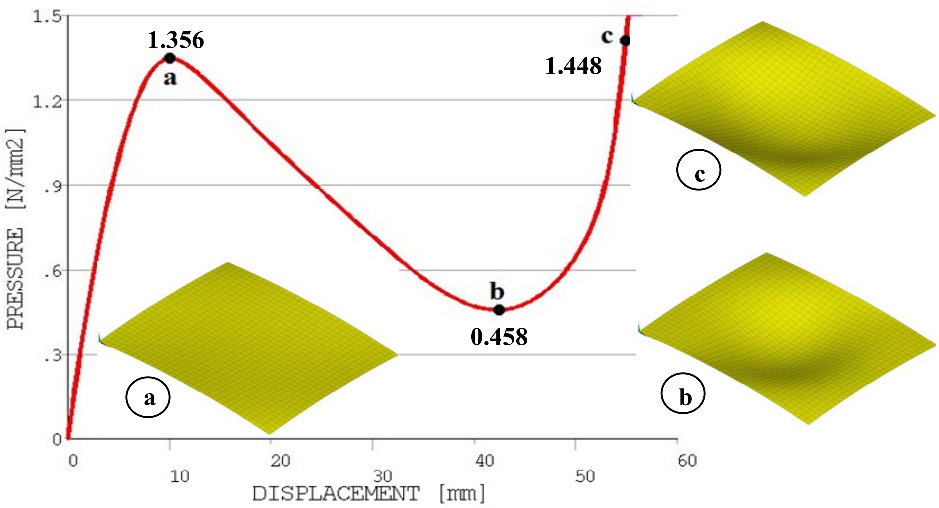

Fig. 3: Fundamental load-displacement path of nonlinear buckling analysis for case 1

Results of nonlinear buckling analysis for ideal shallow shell of translation are presented in Fig. 3. Fundamental load-displacement path for apex node is plotted, values of the load at the limit points are assigned. Shapes of the buckling area are located next to the path.

For next analysis, boundary conditions were changed. Shell of the same dimensions and material properties is supported by hinges only in the corners (case 2). Results from eigenvalue buckling analysis are presented in Fig. 4, the same manner as before. Results from nonlinear buckling analysis follow in Fig. 5.

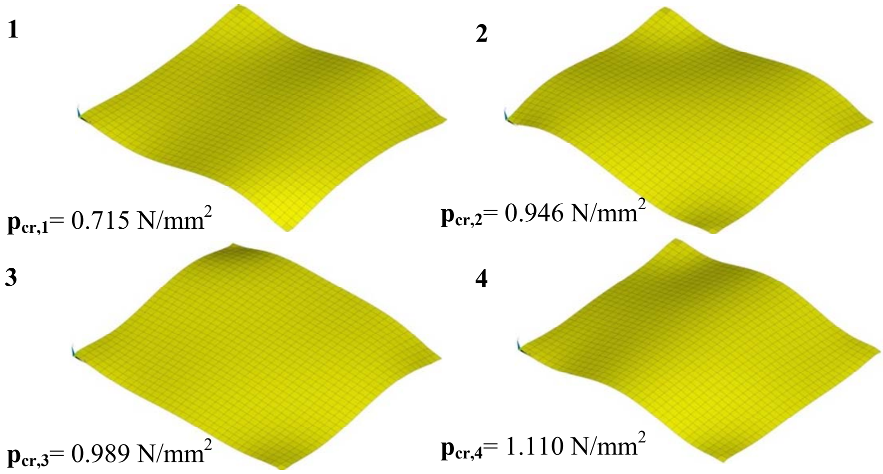

Fig. 4: Results of eigenvalue buckling analysis for case 2

As expected, the difference between the critical load (1st eigenvalue) from eigenvalue buckling analysis (0.715 N/mm^2) and load level in the upper limit point of the load-displacement path of non-linear analysis (0.334 N/mm^2) is significant again.

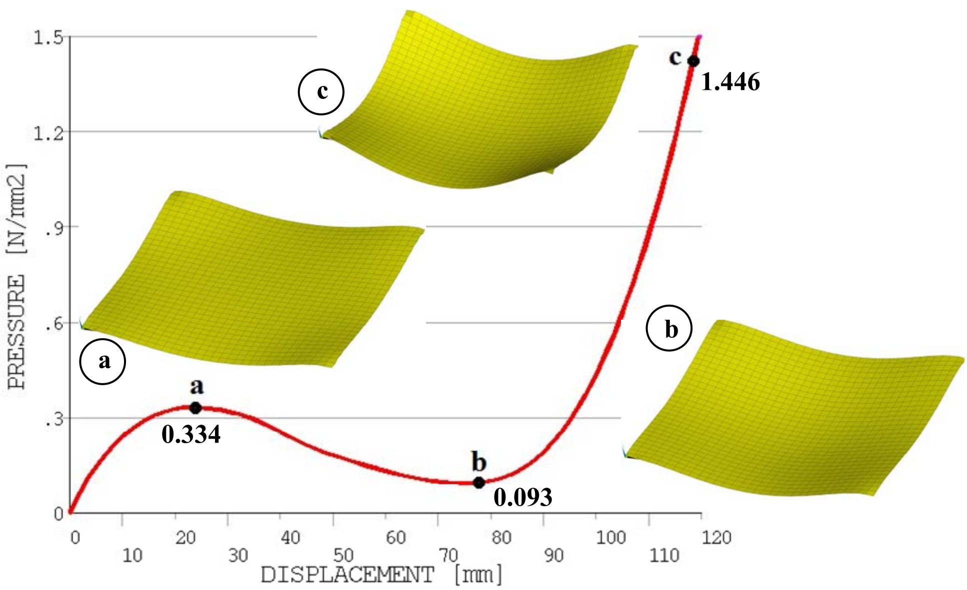

Fig. 5: Fundamental load-displacement path of nonlinear buckling analysis for case 2

Let us now analyze nonlinear solution of imperfect shell. The shape of initial displacements was created identical to a shape of the 1st eigenmode. Multiplier α_0 of the (dimensionless) buckling mode was assumed 0.5 mm and 1 mm respectively.

In Fig. 6 one can observe analysis of case 1 (shell simply supported along the edges). Solution of perfect shell without initial imperfections is plotted by dashed line. Solution of shell with imperfection with magnitude 0.5 mm is plotted by thick line, solution of imperfection with magnitude 1 mm is plotted by thin line. Including the effects of imperfections we can see a further decline of load in the upper limit point in comparison with the perfect shell. In Fig. 7 one can observe analysis of case 2 (shell supported only in the corners).

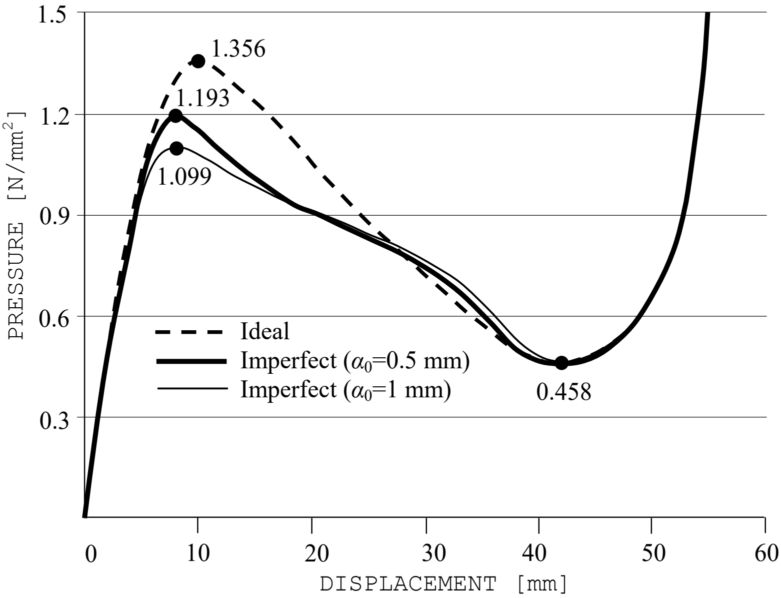

Fig. 6: Fundamental load-displacement path from nonlinear buckling analysis for case 1 (comparison between ideal and imperfect structure)

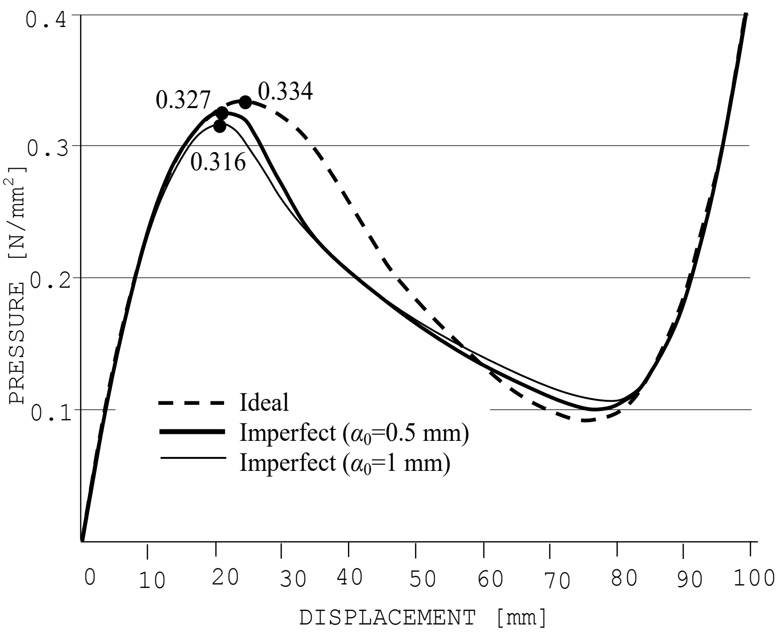

Fig. 7: Fundamental load-displacement path from nonlinear buckling analysis for case 2 (comparison between ideal and imperfect structure)

4 CONCLUSIONS

The analysis of very shallow shells ($h / Ly = 50 / 800 = 1/16$) was presented. Results confirm the fact that the nonlinear approach is necessary. The difference between the critical load of eigenvalue buckling analysis and the load value at the limit point of load-displacement path of nonlinear solution is in the tens of percent (depending on the boundary conditions).

Influence of initial imperfection on the load-displacement path was also investigated. By both Figs. 6 and 7 it can be seen drop of the value of load in limit points with the increase of magnitude of assumed initial imperfection. Hereby the high sensitivity of shallow shells on the mode and magnitude of geometric imperfection was confirmed.

ACKNOWLEDGMENT

Presented results have been arranged due to the research supported by the Slovak Scientific Grant Agency, project No. 1/0272/15. The authors would like to thank for financial contribution from the STU Grant scheme for Support of Young Researchers.

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